

# Technical Notes

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## Computational Analysis of Underexpanded Jets in the Hypersonic Regime

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### I. Introduction

THE phenomenon of underexpanded jets can be found in many engineering applications, such as the aircraft plume, rocket plume, and parallel injection in combustion. Many experiments have been carried out to study this problem.<sup>1-4</sup> Most of these experiments are restricted to underexpanded jets in quiescent ambient flows. Experiments for underexpanded jets in low supersonic freestreams are few, and data for underexpanded jets in hypersonic freestreams are nonexistent.

Because jet plumes contain shocks, expansion waves, and both supersonic and subsonic zones, previous numerical analyses of underexpanded jets often involve complicated viscous/inviscid and subsonic/supersonic coupling schemes. Salas<sup>5</sup> calculated inviscid jet plumes using a shock-fitting method. Dash et al.<sup>6-9</sup> made extensive studies of underexpanded jets using an Euler solver and a parabolized Navier-Stokes solver. These solvers either have to divide the flowfield into domains of different flow properties or have to rely on viscous/inviscid iterations, and their applications are not straightforward. Most of the previous numerical studies emphasis is on jets in quiescent ambience.

In the present work, a full Navier-Stokes equation solver is used to study underexpanded jets. The advantage of the present scheme lies in its simplicity: no viscous/inviscid iteration is needed, and the subsonic and supersonic zones and the jet and the ambient flow are treated uniformly. In order to improve the efficiency of the present scheme, i.e., in order to capture the shock patterns without having to use a large number of grid points, an adaptive grid technique is adopted in the present calculation.

The present study stresses jets in supersonic and hypersonic ambient flows. The interaction between the jet and the supersonic ambient flow is studied in detail. It is found that this in-

teraction considerably alters the jet structure. Because it is difficult to collect experimental data, little is known about jet flows at hypersonic conditions; the present work studies the characteristics of hypersonic jet flows numerically and attempts to obtain information that is difficult to produce experimentally.

### II. Mathematical and Numerical Formulations

#### A. Equations and Solver

The full Navier-Stokes equations in conservation law form are solved in the present study. An existing computer code, PARC, originally developed by Pulliam et al.<sup>10</sup> as ARC and later modified by the AEDC Group of Sverdrup Technology,<sup>11</sup> is used in the present study. This code solves the Navier-Stokes equations using the approximate factorization algorithm originally developed by Beam and Warming. Central difference is used to discretize the spatial derivatives, and backward difference is used for the time derivatives. To avoid having to solve a block pentadiagonal matrix, the Jacobian matrices are diagonalized using their eigenvalues and eigenvectors. This procedure results in a set of scalar pentadiagonal equations. Second-order and fourth-order artificial dissipation terms are used in the code to ensure stability and convergence. The axisymmetric version of the code is used in this study.

#### B. Grid

An underexpanded jet contains a complex shock pattern, and an accurate solution of the flowfield relies on an accurate capturing of the shocks. Our experience has been that, with a grid of manageable size, grid adaptation is essential for accurate shock capturing. This is especially true for the case of underexpanded jet into quiescent flow, where multishock cells exist. A simple adaptive grid scheme based on the arc equidistribution concept is used in the present work for shock capturing. The details of this scheme are documented in Ref. 12. In this scheme, the grid size is required to be inversely proportional to the gradient of flow variables. A filter type of smoothing scheme is devised to filter out the high-frequency oscillations in the grid.

### III. Numerical Analysis of Jet Flows

Several different jet Mach numbers  $M_j$ , ambient Mach numbers  $M_a$ , and pressure ratios  $p_j/p_a$  have been considered in the present study. These flow conditions are given in Table 1.

#### A. Mach 2 Jet in a Quiescent Ambience

It is known that compressible flow solvers often have difficulties in dealing with very low Mach number flows. According to Chuech,<sup>3</sup> a coflow must be added to stabilize the solution when using Dash's SCIPVIS code to solve jet flows in a quiescent environment. Our experience shows that changes in coflow can change the cell length in the jet considerably. In the present study, a jet flow into a true quiescent environment is studied. Convergence of the solution is improved by reducing the size of the zero Mach number region. Converged solutions are obtained for the relatively low jet-to-ambient pressure ratio  $p_j/p_a = 1.45$  with a jet Mach number  $M_j = 2$ . A uni-

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form grid of  $200 \times 80$  was used for the initial calculation. The grid is then adapted to the flowfield: pressure is used to construct the weight function for the  $x$  direction and the  $u$  velocity is used for the  $y$  direction. The use of flow-adaptive grids enables one to capture the shocks and the shear layer better, thus improving the accuracy of the predicted pressure. This is illustrated by the calculated jet centerline pressure as compared with Seiner's experimental data<sup>1</sup> in Fig. 1.

### B. Mach 3 Jet in a Mach 2 Freestream

Because underexpanded jets find most of their applications in supersonic flights, the study of jets in supersonic ambient flows is more important than the study of jets in still air. In the present work, a Mach 3 underexpanded jet in a Mach 2 freestream is studied. Three jet-to-ambient pressure ratios,  $p_j/p_a = 2, 5$ , and  $10$ , are considered.

The Mach number contours for the case  $p_j/p_a = 5$  are given in Fig. 2. One can see that the jet structure and the ambient

flow pattern are considerably different from those of the case with an ambience of still air. When the ambient flow is quiescent, the jet boundary is that of a constant pressure. All of the waves are reflected on this boundary and become waves of the opposite sense (e.g., compression waves are reflected as expansion waves). This is the mechanism that causes a series of shock cells in the underexpanded jet; these shock cells are manifested by the pressure oscillation shown in Fig. 1. However, when the ambient flow is supersonic, the jet boundary is no longer a boundary of constant pressure. Because the jet expands and the freestream changes direction, an oblique shock develops in the ambient flowfield at the jet exit, which increases the pressure at the jet boundary and essentially decreases the effective jet-to-ambient pressure ratio. The overexpansion is significantly reduced when the turning jet boundary induces a series of expansion waves in the ambience, which can also be interpreted as waves in the jet transmitted through the jet boundary. When the shock from the Mach disk meets the jet boundary, it helps the jet flow adjust to the direction of the freestream. There is an oblique shock in the freestream originating from the point where the jet boundary turns to the freestream direction. Again, this shock can be explained either as arising from the turning of the jet boundary or as a shock from the jet transmitted through the boundary. The above described process helps to adjust the jet pressure to that of the ambience very quickly, normally within one shock cell, although the cell lengths are much longer here than in the still air cases. This mechanism of adjusting the jet pressure to the ambient pressure is what we call the supersonic pressure relief effect.

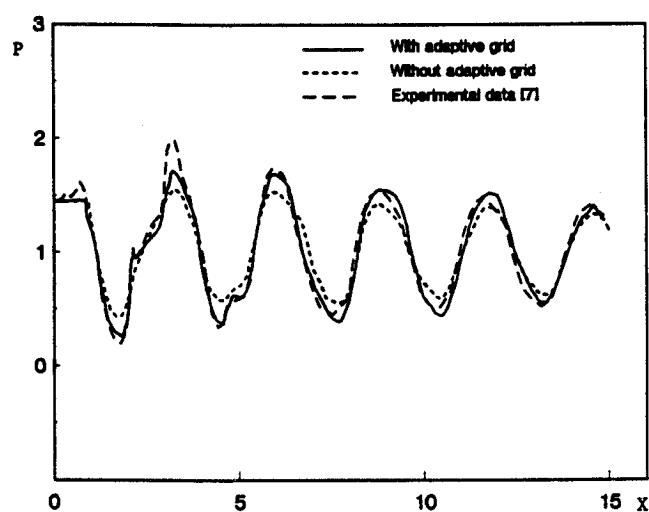


Fig. 1 Jet centerline pressure distributions for a Mach 2 jet in still air,  $p_j/p_a = 1.45$ .

Table 1 Flow conditions

Case	$M_j$	$M_a$	$p_j/p_a$
1	2	0	1.45
2	3	2	2
3	3	2	5
4	3	2	10
5	6	5	3
6	6	5	5
7	6	5	10

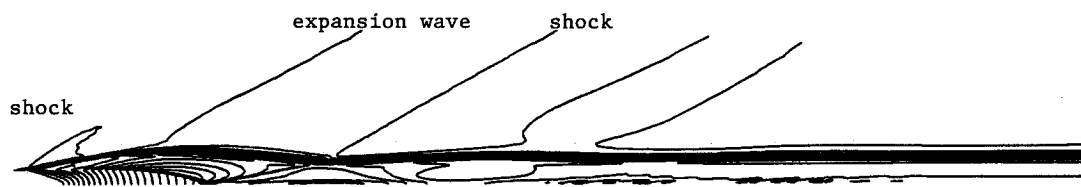


Fig. 2 Mach number contours of a Mach 3 jet in a Mach 2 ambient flow,  $p_j/p_a = 5$ .

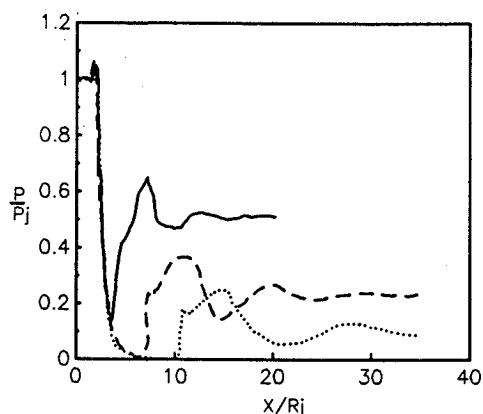


Fig. 3 Centerline pressure distributions for underexpanded jets in a supersonic ambient flow;  $M_j = 3$ ,  $M_a = 2$ ; —  $p_j/p_a = 2$ ; ---  $p_j/p_a = 5$ ; .....  $p_j/p_a = 10$ .

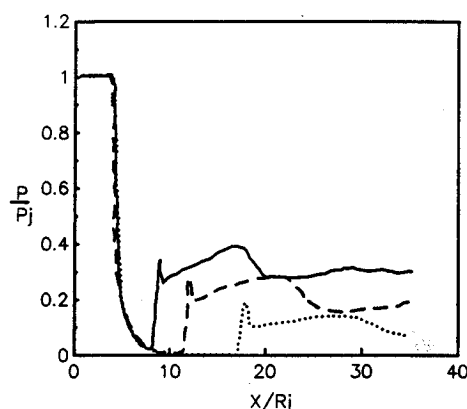


Fig. 4 Centerline pressure distributions for underexpanded jets in a hypersonic ambient flow;  $M_j = 6$ ,  $M_a = 5$ ; —  $p_j/p_a = 3$ ; ---  $p_j/p_a = 5$ ; .....  $p_j/p_a = 10$ .

The jet centerline pressures for all three supersonic flow cases are given in Fig. 3. The centerline pressure distributions show that, in all three cases, the pressure oscillations diminish rapidly after the first shock cell.

#### C. Mach 6 Jet in a Mach 5 Freestream

The primary goal of the present work is to understand the structures of hypersonic jets. Toward this end, a Mach 6 jet in a Mach 5 freestream is studied. Three jet-to-ambient pressure ratios are considered; they are  $p_j/p_a = 3, 5$ , and 10.

The present results show that the basic structure of a hypersonic jet is similar to that of a supersonic jet. All of the previous analyses about supersonic jets also apply here. The major distinct features for jets in the hypersonic regime are that the cell lengths are much longer, the shocks and expansion waves induced in the freestream by the jet stay fairly close to the boundary of the jet, and, therefore, the exit shock tends to merge with the shear layer of the jet boundary, thus making it a layer of rapid changing pressure and rapid changing velocity.

The centerline pressure distributions for the three hypersonic jet cases mentioned earlier are given in Fig. 4. A comparison between Figs. 3 and 4 shows that as the flight Mach number increases, the oscillation in the centerline pressure behind the first cell decreases.

#### IV. Concluding Remarks

The present numerical study demonstrates that the phenomenon of underexpanded jets can be analyzed using full Navier-Stokes solvers without resorting to special treatments of the jet flowfield. It also shows that the jet structures are considerably different for subsonic and supersonic ambient flows. A supersonic pressure relief effect has been identified. This pressure relief effect reduces the pressure oscillation in an underexpanded jet in supersonic ambient flows. The study also shows that the length scales in hypersonic jets are considerably different from those in supersonic jets.

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## Propellant Feed System of a Regeneratively Cooled Scramjet

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#### Nomenclature

$F$	= thrust, N
$h$	= height of the inlet
$I_{sp}$	= specific impulse, $\text{m} \cdot \text{s}^{-1}$
$M$	= Mach number
$\dot{m}$	= mass flow rate, $\text{kg} \cdot \text{s}^{-1}$

#### Subscripts

$cl$	= coolant
$H_2$	= hydrogen
$st$	= stoichiometric
$\infty$	= freestream

#### I. Introduction

THERE is great interest in the use of scramjets as engines for aerospace planes, but there have been few reports<sup>1-3</sup> on scramjet engine cycles, in other words, propellant supply systems. Therefore, the feasibility of scramjet engine cycles has not been made clear, and potential problems in scramjet development remain unclear. To address this deficiency, a cycle analysis has been conducted of an airframe-integrated hydrogen-fueled scramjet engine.

The flight path of the vehicle is along a constant dynamic pressure of 100 kPa. The range of flight Mach numbers is from 6-12.

#### II. Assumptions and Methods of Calculation

##### A. Engine Cycle Schematic and Configuration

An expander cycle is adopted as a fuel supply system (see Fig. 1). The turbine is driven by gaseous hydrogen that is heated while regeneratively cooling the engine. The six scramjet engine modules are mounted on a vehicle. Each component of the air breathing part, e.g., the combustor, is cooled individually. The frontal area of the inlet of each module is  $1 \text{ m}^2$ . The length from the entrance of the inlet to the exit of the inner nozzle is 7.7 m.

In principle, the equivalence ratio in the combustor is kept at unity. When the hydrogen flow rate required for engine cooling exceeds the stoichiometric flow rate, all of the hydrogen required for cooling is injected into the combustor. The engine overall thrust and the specific impulse with stoichiometric combustion are referred to as the reference values in the discussion.

##### B. Properties of Combustion Gas

The temperature, pressure, speed, and specific impulse in the air breathing part are calculated using a quasi-one-dimensional model with friction, combustion, and heat transfer.<sup>4</sup> In the model, the following methods and assumptions are used:

1) The vehicle forebody is approximated by a cone. The results with the cone's half-angle of 5 deg are referred to here.

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